Neural Symbolic computing

Article review

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## Abstract

Neuro-symbolic computing is an area of AI research which seeks to combine traditional rules-based AI approaches with modern deep learning techniques. In recent years, neural systems have displayed highly effective learning ability and superior perception intelligence, but have been found to lack cognitive ability with effective reasoning. In contrast, symbolic systems have exceptional cognitive intelligence, but their learning capabilities are poor compared to neural systems. Considering the advantages and disadvantages of both methodologies, an ideal solution is to combine neural systems and symbolic systems, an approach that produces neural-symbolic systems with powerful perception and cognition. In this paper we tried to review researches that offer a systematic overview of the important and recent developments on Neuro-symbolic computing. We looked at papers that study the history of this area, covering early work and foundations. We further looked at researches that discuss background concepts and identify key driving factors behind the development of Neuro-symbolic computing. In addition, we reviewed recent landmark approaches along several main characteristics that underline this research paradigm, including neural-symbolic integration, knowledge representation, knowledge embedding, and functionality. Then, we looked at researches that discuss the successful application of modern Neuro-symbolic computing approaches in several domains. Finally, we identify open problems together with potential future research directions.

## Introduction

Neural-symbolic computing aims at integrating two most fundamental cognitive abilities: the ability to learn from experience, and the ability to reason from what has been learned. The integration of learning and reasoning through neural-symbolic computing has been an active branch of AI research for several years. Neural-symbolic computing aims at reconciling the dominating symbolic and connectionist paradigms of AI under a principled foundation(Garcez et al., 2019).

The integration of learning and reasoning is one of the key challenges in artificial intelligence and machine learning today, and various communities have been addressing it. That is especially true for the field of neural-symbolic computation, where the goal is to integrate symbolic reasoning and neural networks. Neural-symbolic computation already has a long tradition, and it has recently attracted a lot of attention from various communities(Marra et al., 2021).

Neural networks have traditionally been grey-box systems. It’s trivial to observe the internal physical structure of a network, since a model’s topology is explicitly defined as part of hyper parameter selection. However, mapping the high dimensional, abstract features which neural networks manipulate to concrete concepts which humans can understand has proven to be difficult. Such mappings are highly desirable for safety-critical fields such as medicine and self-driving cars, where we must be certain a model is learning the desired dataset features rather than over-fitting to data artifacts. Networks composed of independent sub models have an advantage here, since each sub model was trained for a specific purpose. Knowing the purpose of each sub model gives us some information about the internal state of the top-level model(Susskind et al., 2021).

The last years have been characterized by an upsurge of opaque decision systems, such as Deep Neural Networks (DNN). Although they have great generalization and prediction skills, their functioning does not allow detailed explanations of their behavior. As opaque machine learning models are increasingly being employed to make important predictions in critical environments, the danger is to create and use decisions that are not justifiable or legitimate(Bennetot et al., 2019).

Recent years have witnessed the development of neural-symbolic computing, which enables more and more applications to emerge. We discuss some popular applications and tasks to stimulate innovations of neural-symbolic computing in more application areas in the future such as scientific discovery which requires algorithms to respect real world constraints and can be interpreted and understood by scientists, an important approach for explainable AI, as such, neural-symbolic computing is naturally suitable for scientific discovery. Another is programming systems which the AI community has shown great interest in developing machine learning assistants for programming systems. However, it is challenging for current neural-based AI to understand the syntactic and semantic constraints for program synthesis whereas neural-symbolic AI incorporates the advantage of the symbolic methods that give consideration to both high-level reasoning and low-level implementation, which is naturally suitable for program generation. Yet another one is mathematical reasoning a critical skill for human beings, mathematics poses a particularly attractive learning challenge for AI since it is the premise of general purpose reasoning. However, early neural network-based systems are limited to the simple form of mathematical reasoning, such as arithmetic tasks that only involve integer addition and multiplication. Moreover, their fully sub-symbolic nature makes them not mathematically interpretable. Considering that symbols are the most common objects in mathematical reasoning, it is a natural choice to adopt neural-symbolic AI for mathematical reasoning (Wang et al., n.d.).

## Objective

The objective of this review paper is to offer a systematic overview of the important and recent developments on neuro-symbolic computing.

## Specific Objectives

* Review papers that study the history of this area, covering early work and foundations.
* Look at researches that discuss background concepts and identify key driving factors behind the development of neuro-symbolic computing.
* Review recent landmark approaches along several main characteristics that underline this research paradigm, including neural-symbolic integration, knowledge representation, knowledge embedding, and functionality.
* Discuss the successful application of modern neuro-symbolic computing approaches in several domains.
* Identify open problems together with potential future research directions.

## State of the Art

While researchers have been making great progress in AI, they still haven’t been able to give machines the special ingredient that makes us ‘us’: common sense. We just know, seeing a person walk in soaking wet that it’s raining outside. Today, machines translate languages, recognize objects and spoken speech. But ask a smartphone assistant something more complex than a basic command, and it will struggle. Machines with common sense, which rely on an emerging AI technique known as neurosymbolic AI, could greatly increase the value of AI for businesses and society at large. Such AI would also require far less training data and manual annotation, as supervised learning consumes a lot of data and energy — to the point that if we keep on our current path of computing growth, by 2040 we’ll exceed the ‘power budget’ of the Earth. There’s simply not enough data or power to continue on with today’s AI.

Go back to mid-1950, with the coining of the term “artificial intelligence” in1955. The field really kicked off the year after. Back then, the approach to AI resorted to symbols representing objects and actions, similar to how humans process information. That’s the essence of the once-mainstream approach to machine learning called symbolic AI that is still used today, albeit not very widely. It is based on the idea that humans make sense of the world by creating internal symbolic representations and rules for dealing with them, based on logic. These rules can be turned into a computer algorithm, capturing our daily knowledge — describing, for instance, that if a ball is thrown and there is no wall, it should keep going straight. But if there is a wall, it should bounce back. The computer uses these structured representations of knowledge and applies logic to manipulate them, gaining new knowledge to ‘reason’ somewhat similar to humans.

The 1990s saw major developments through really powerful tools of probabilistic models and statistical inference, the paradigm that gave us the modern field of machine learning. This probabilistic approach led to advances in natural language processing and machine learning, and drove technologies we now take for granted.

The development of deep learning triggered an AI boom and the field exploded around 2012. But deep learning isn’t without its limitations. One significant challenge is that neural nets can’t explain how objects relate to each other. As they rely on available data, they can’t reason — they can’t have common sense. Common sense is all of the implicit knowledge that we have that’s never written down anywhere.

Furthermore, the ongoing revolution in artificial intelligence (AI)—in image recognition, natural language processing and translation, and much more—has been driven by neural networks, specifically many-layer versions known as deep learning. These systems have well-known weaknesses, but their capability continues to grow, even as they demand ever more data and energy. At the same time, other critical applications need much more than just powerful pattern recognition, and deep learning does not provide the sorts of performance guarantees that are customary in computer science.

To address these issues, some researchers favor combining neural networks with older tools for artificial intelligence. In particular, neurosymbolic AI incorporates the long-studied symbolic representation of objects and their relationships.

Machines have been trying to mimic the human brain for decades. But neither the original, symbolic AI that dominated machine learning research until the late 1980s nor its younger cousin, deep learning, have been able to fully simulate the intelligence it’s capable of. One promising approach towards this more general AI is in combining neural networks with symbolic AI.

## Discussion

In the paper “Neural-Symbolic Computing: An Effective Methodology for Principled Integration of Machine Learning and Reasoning” (Garcez et al., 2019) the authors presented the principles of neural-symbolic integration by highlighting key characteristics that underline this research paradigm. Despite their differences, both the symbolic and connectionist paradigms, share common characteristics offering benefits when integrated in a principled way. For instance, neural learning and inference under uncertainty may address the brittleness of symbolic systems. On the other hand, symbolism provides additional knowledge for learning which may ameliorate neural network’s well-known catastrophic forgetting or difficulty with extrapolating. In addition, the integration of neural models with logic-based symbolic models provides an AI system capable of bridging lower-level information processing (for perception and pattern recognition) and higher-level abstract knowledge (for reasoning and explanation). They reviewed the important and recent developments of research on neural-symbolic systems by outlining the main important characteristics of a neural-symbolic system: Representation, Extraction, Reasoning and Learning, and their applications. They then discussed and categorized the approaches to representing symbolic knowledge in neural-symbolic systems into three main groups: rule-based, formula-based and embedding-based. After that, they showed the capabilities and applications of neural-symbolic systems for learning, reasoning, and explainability. At the end of the paper they discussed recent trends and identified a few challenges for neural-symbolic computing research.

The paper also highlighted the key ideas and principles of neural-symbolic computing. In order to do so, they illustrated the main methodological approaches which allow for the integration of effective neural learning with sound symbolic-based, knowledge representation and reasoning methods. One of the principles they highlighted in the paper is the sound mapping between symbolic rules and neural networks provided by neural-symbolic computing methods. This mapping allows several knowledge representation formalisms to be used as background knowledge for potentially large-scale learning and efficient reasoning. They showed that the interplay between efficient neural learning and symbolic reasoning opens relevant possibilities towards richer intelligent systems. The comprehensibility and compositionality of neural-symbolic systems, offered by building networks with a logical structure, allows for integrated learning and reasoning under different logical systems.

The key motivation behind the paper “From Statistical Relational to Neural Symbolic Artificial Intelligence: a Survey” (Marra et al., 2021) is its aim at pointing out the similarities between StarAI and NeSy. In a way it wants to stimulate cross-fertilization, and to doing so, it starts from the literature on StarAI, because it turns out that the same issues and techniques that arise in StarAI apply to NeSy as well. This survey identified seven dimensions that these fields have in common and that can be used to categorize both StarAI and NeSy approaches. These seven dimensions are concerned with (1) type of logic, (2) model vs proof-based inference, (3) directed vs undirected models, (4) logical semantics, (5) learning parameters or structure, (6) representing entities as symbols or sub-symbols, and (7) integrating logic with probability and/or neural computation. The first seven sections of the paper describes one dimension each by introducing the main concepts first, either based on logic, probability or machine learning, and then showing how they are implemented in StarAI and NeSy systems. The first two sections deal with purely logical dimensions: Section 2 introduces the different types of logic, i.e. propositional, relational and first-order, while Section 3 presents how to use logic for inference by distinguishing between proof-based and model-based systems. Section 4 distinguishes between directed or undirected models, as it is common in probabilistic graphical models. Section 5 presents how Boolean logic can be extended to continuous semantics, namely probabilistic and fuzzy logic. Section 6 introduces the dimension of learning, distinguishing parameter learning from structure learning. Section 7 focuses on representations and to what extent neural symbolic models use symbolic and/or sub symbolic features. Section 8 positions neural symbolic approaches in the spectrum of three main paradigms, i.e. logic, probability and neural networks. Section 9 proposes inter-dimensional considerations. Section 10 analyses two deep learning techniques, namely knowledge graph embedding and graph neural networks, in the spirit of the proposed dimensions, as it turns out that they share many features with neural symbolic systems.

The paper “Neuro-Symbolic AI: An Emerging Class of AI Workloads and their Characterization” (Susskind et al., 2021) analyzes the inference performance characteristics of three separate neuro-symbolic models. Two of the models, the Neuro-Symbolic Concept Learner (NSCL) and Neuro-Symbolic Dynamic Reasoning (NS-DR), are composed of multiple independent ”sub models”, which extract problem features before passing them as input to a final symbolic sub model. The third, Neural Logic Machines (NLM), is an end-to-end model without independent sub models. The NLM model uses an object’s relations, properties, quantifiers, and logic connectives in order to accomplish the task of generalization.

The paper is organized in sections where: section2 outlines the space of neuro-symbolic learning, including the progressive improvements to neuro-symbolic models. Section 3 describes the three models that we are analyzing in this paper in detail. Section 4 describes the methodology for analyzing the performance of these models, based on classifying activity into eight distinct categories. Section 5 provides the results of the analysis with breakdowns for each sub model component. As a result, while neuro-symbolic models with sub models look topologically distinct from traditional deep learning models, the analysis suggests that their performance characteristics can be largely viewed as a combination of existing workloads. In addition, the analysis of the NSCL and NS-DR show that there are relatively few opportunities for acceleration of symbolic computation. The symbolic workloads of these two models have low operational intensities, consisting of vector and/or scalar operations, and exhibit complex control flow. These factors combined greatly limit the potential for parallelism. However, the symbolic components do not make up large portions of the execution times of either workload, and are therefore unlikely to pose a bottleneck. On the other hand, the NLP, dynamics predictor, and vision sub models exhibit different dominant operation categories, including dense GEMM in the question parser, data movement in the dynamics predictor, and a co-dominance of convolution and element-wise operations in the image and frame parser. NLM models require numerous element-wise operations for inference, with the computational demand increasing with the complexity of the task-specific logical rule set. The challenge of accelerating the low-operational-intensity, element-wise computations of workloads such as NLM will become increasingly important as this field sees further development.

The survey “Recent Advances in Neural-symbolic Systems: A Survey” (Yu et al., 2021) summarize the characteristics, the advantages and disadvantages of symbolic systems and neural systems respectively. It state that symbolic systems make good use of knowledge, while neural systems make good use of raw data; in short, they complement. Therefore, neural-symbolic systems are a good choice for scenarios in which the availability of training data is limited, or the model would benefit from more interpretability. The paper provides an analysis of neural-symbolic systems from three perspectives: efficiency, generalization, and interpretability. In terms of efficiency, neural-symbolic systems can reasoning rapidly, meaning that they can reduce exponential computational complexity to polynomial complexity. The powerful computation ability of neural networks can be attributed to this improved efficiency. The paper summarize two principles underpinnings of learning for reasoning approaches with a particular focus on the way in which a neural network should be integrated. (1) Accelerator. Symbolic reasoning technologies need to search large spaces to arrive at an expected solution. To accelerate solution search process, neural networks can replace traditional search techniques with an accelerator, such as reinforcement learning, etc. (2) Transformer. Symbolic systems need to take symbols as input. When dealing with unstructured data, neural networks can map these data into symbols to facilitate their use as input for symbolic reasoning. In addition, it also summarize the following key factors in reasoning for learning. (1) Knowledge representation. Symbolic knowledge is a kind of discrete representations. Most methods of combining symbolic knowledge with neural networks opt to convert the symbolic knowledge into an intermediate representation, such as a graph, to obtain a continuous representation. Moreover, some approaches use fuzzy logic such as t-norm to assign soft truth degrees in the continuous set. (2) Combining approaches. One type of approach involves taking symbolic knowledge as a regularization term in the loss of the neural networks. The others involve integrating symbolic knowledge into the structure of the neural networks to improve their performance. Furthermore, the paper discusses Learning-reasoning approaches are increasingly popular in AI research, as they enjoy the advantages of both neural networks and symbolic reasoning: specifically, the neural networks provide facts with symbolic reasoning, and the symbolic reasoning constrains/helps learning of the neural networks. For instance, DeepProbLog and ABL have similar model principles: that is, the modeling of complex problems is defined in logic programming language, and the neural network is used to define simple concepts in logic programming language. These operate in a unified framework in neural-symbolic systems. BPGR uses neural networks to accelerate the search process of symbolic reasoning, along with symbolic knowledge to constrain neural network learning. This model not only characterizes the degree of matching, but also clearly states which symbolic knowledge is being fitted, along with the probability of this symbolic knowledge being fitted as an explanation for the model prediction. Based on representative works of the methods in the three categories discussed above, the paper summarize a general design idea for neural-symbolic approaches. The iterative loop between neural networks and symbolic systems allows embedding of symbolic knowledge into neural network models, as well as the learned symbols from neural networks into symbolic systems. It also summarize the characteristics that should be considered in designing neural-symbolic approaches, as follows: (1) Uncertainty. The output of the neural network is a distribution, not “true” or “false”. This results in a need to consider the uncertainty of triggered symbolic knowledge. (2) Globalization. It is necessary to consider the fit of all symbolic knowledge in the knowledge base, not just the local knowledge. (3) Importance. Different knowledge may have different weights, and the degree of fitting knowledge with different weights should be considered. (4) Interpretability. Interpretability should be explicitly considered in learning (in terms of e.g. the immediate process of learning of the result of learning).

In the paper “A Neural-Symbolic Approach to Natural Language Understanding” (Liu et al., 2022) the authors discusses how deep neural networks, empowered by pre-trained language models, have achieved remarkable results in natural language understanding (NLU) tasks. However, their performances can drastically deteriorate when logical reasoning is needed. This is because NLU in principle depends on not only analogical reasoning, which deep neural networks are good at, but also logical reasoning. According to the dual-process theory, analogical reasoning and logical reasoning are respectively carried out by System 1 and System 2 in the human brain. Inspired by the theory, the paper present a novel framework for NLU called Neural-Symbolic Processor (NSP), which performs analogical reasoning based on neural processing and logical reasoning based on both neural and symbolic processing. As a case study, they conduct experiments on two NLU tasks, question answering (QA) and natural language inference (NLI), when numerical reasoning (a type of logical reasoning) is necessary. The experimental results show that their method significantly outperforms state-of-the-art methods in both tasks. The paper proposes a novel framework for natural language understanding (NLU), referred to as Neural-Symbolic Processor (NSP). NSP employs two types of reasoning, analogical reasoning and logical reasoning. To ‘understand’ language, analogical reasoning is performed by using neural networks as usual. In the meantime, logical reasoning is performed by using neural networks to generate programs and then using symbolic systems to execute the programs. Such an architecture is similar to that of humans, and the two types of reasoning correspond to System 1 and System 2 respectively in the human brain. The paper’s approach thus is powerful in dealing with the challenging problems which conventional neural-network-only approaches suffer from. They evaluate their approach in two NLU tasks, QA and NLI. The experiments show that their method surpasses previous methods with remarkable improvements when logical reasoning is needed. Although NSP outperforms the baselines in both QA and NLI, there are still complicated cases that the current NSP cannot effectively deal with. For the QA task, they conduct an error analysis of NSP on the DROP dataset. They provided three typical categories of hard cases for NSP. The first type (first example) is related to multi-step reasoning. Such cases need a deep reasoning path or many arguments in functions. Another type (second example) is about the counting of long strings. Defining a standard for program annotation is hard because different human annotators may select strings with different lengths. The last type (third example) is related to complex conditions. The programs of NSP used a simple grammar language, which still cannot represent complicated conditions in reasoning. They also conduct an error analysis of NSP for the NLI task.

The paper “Towards Data-and Knowledge-Driven AI: A Survey on Neuro-Symbolic Computing” (Wang et al., n.d.) Offers a systematical and timely collection of recent important literature on NeSy, with particular attention to the past five years. The surveyed papers are those works published in the flagship repositories for machine learning and related areas, such as computer vision, natural language processing (NLP), and knowledge graph, or have been widely cited. The survey offers an exhaustive and up-to-date literature overview to researchers of interest, and nourish the exploration of open and developmental issues. They also remark that the survey is undoubtedly a biased view, as there is a broad spectrum of research in this fast-growing area, but they do attempt to identify and analyze common and critical properties of landmark practices in order to cover the major research threads. The structure of the paper is as follows: In Section 2, they begin with a brief review of early research results of NeSy, which shape the latest effort in this area. Section 3 then elaborates on the general concepts of mind in psychology and cognitive science, which underpin the theoretical foundations of NeSy; and discusses recent debate regarding the necessary and sufficient building blocks of AI, which promotes the advance of this area. In Section 4, they state their taxonomy of NeSy, which classifies recent important NeSy literature according to four dimensions, namely neural-symbolic interrelation, knowledge representation, knowledge embedding, and functionality. In Section 5, they elaborate on popular and emerging application areas of NeSy. Finally, Sections 6 and 7 present potential valuable directions for further research and conclude the survey. This survey will help newcomers and practitioners to navigate in this massive field which gained significant momentum in the past few years, as well as provide AI community background in generating future research.

## Conclusion and Future Work

Though recent years have witnessed significant progress in the research of neuro-symbolic computing, there still exist many open problems that are worth exploring. Scalability: Current neuro-symbolic computing researches with an explicit logical or probabilistic reasoning modeling are often limited by rapidly increasing hardness as more complex logics required. One reason is of the growing computational complexity proportional to the expressivity of logic rules, e.g., the inclusion of universal quantification over variables. On the other hand, the frequent reliance on hand crafted rules or domain specific knowledge also impedes their scalability to large-scale applications in the wild. Owing to the superiority of connectionist components that learn from data at scale, neuro-symbolic computing is expected to have sufficient capacity for the full use of rich data and handling real-world challenges. However, whether it is possible to scale to the real-world complexity, without sacrificing desirable features from the symbolic aspect, remains unknown and deserves further investigations. Compositional Generalization: Compositionality, a core of human intelligence, is among the desirable characterizations where the neuro-symbolic computing systems are expected to be good at. It allows strong generalizability to novel reasoning problems, through decomposing and recombining the learned knowledge, in terms of symbolic representations or neural modules. Along this line, state-of-the-art researches are mainly conducted on discrete symbols, i.e., natural language word tokens, whether it is possible, or to what extent, to achieve the high-level composition, such as incorporating different types of logic systems (e.g., modal, temporal, common sense, epistemic, etc.), remains an open challenge. Neuro-symbolic systems are still seeking to have the realistic property of compositionality. Automatic and Comprehensive Symbolic Knowledge Acquisition: Even though favorable progress has been made in symbolic knowledge acquisition, much of the work is studied on the non-logical symbolic level, particularly in form of conceptual knowledge graphs, and largely restricted to sophisticated learning algorithms that often do not scale well to deep networks. Furthermore, incorporating complex logic, probabilistic relation or various data sources inevitably complicates the problem even more. Some believe that more emphasis is needed on comprehensively and automatically discovering symbolic knowledge from not only the data of growing scale, but also networks with explosive dimensionality. Recursive Neuro [Symbolic] Engine: Yet another invaluable research endeavor is to make progress on an automated Neuro [Symbolic] engine that is able to benefit from the interplay between the neural- and symbolic components. Primarily, the engine should be capable of combinatorial reasoning towards a level bordering the human intelligence. Besides, the neural components are designed to be trained against the useful symbolic constraints, while recursively, the symbolic components are in turn able to evolve with the higher-quality rules induced from data. This loop is closely linked to the knowledge acquisition challenge that was discussed. It opens several promising research lines that show strong potential of (fundamentally) addressing the scalability issue, and also offers a rich alternative towards a formal realization of System 1 & System 2. Testbed for Metacognitive Skills of Neuro-symbolic system: There is a strong agreement that neuro-symbolic systems are among the most promising avenues towards human-like AI. However, the main strands of its applications are still restricted by a handful of tasks that typically focus on only basic decision making and reasoning problems, in contrast to the large vision researchers hold onto the metacognitive capabilities of human beings, such as productivity, systematicity, compositionality and inferential coherence of mental thought, causal and counterfactual thinking, deductive reasoning, interpretability, etc.

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